

Multi-loop Constrained Iterative Model Predictive Control Using ARX -PLS Decoupling Structure^{*}

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Abstract A multi-loop constrained model predictive control scheme based on autoregressive exogenous-partial least squares (ARX-PLS) framework is proposed to tackle the high dimension, coupled and constraints problems in industry processes due to safety limitation, environmental regulations, consumer specifications and physical restriction. ARX-PLS decoupling character enables to turn the multivariable model predictive control (MPC) controller design in original space into the multi-loop single input single output (SISO) MPC controllers design in latent space. An idea of iterative method is applied to decouple the constraints latent variables in PLS framework and recursive least square is introduced to identify ARX-PLS model. This algorithm is applied to a non-square simulation system and a stirred reactor for ethylene polymerizations comparing with adaptive internal model control (IMC) method based on ARX-PLS framework. Its application has shown that this method outperforms adaptive IMC method based on ARX-PLS framework to some extent.

Keywords partial least square, constraint, model predictive control, iterative method

1 INTRODUCTION

As a kind of advanced control scheme, model predictive control has gained widely application in both academic and industrial area during the past three decades. There are many successful cases of predictive control in including those of robots [1], clinical anesthesia [2], drying towers [3], distillation columns [4] and steam generators [5]. The good performance of these applications shows the capacity of MPC to achieve highly efficient control performance. This is due to its ability to deal with (1) multivariable (square or non-square) systems, and (2) systems with hard and soft constraints using simple and intuitive process descriptions such as transfer function [6]. Operation at or near equipment limitation or safe constraints is necessary for the most profitable or most efficient operation in many cases [7]. The most significant priority of MPC to other traditional controller is its ability to handle such constraints, especially on controls and states [8]. However, the traditional MPC also has its drawbacks. Firstly, when system models has higher dimensionality, it will result in not only the complexity of the controller but also the huge computation of control law solving [9]. Secondly, the complex collinearity of processes variables, manipulated variables and the complex collinearity, for example, enhance the complication of MPC controller. Therefore, it is necessary to find an effective method in order to overcome those disadvantages in traditional MPC scheme.

The partial least square regression method (PLS) [10, 11] has the potential capacity to overcome the proposed drawbacks of MPC scheme. PLS is a

well-known decoupling scheme that decomposes the original multivariable system into several subspaces, though construct several new latent variables orthogonal to each other. Many methods using PLS offer data compression facility to condense the variance of process data into low-dimension latent subspaces in chemical and other industries [12–14]. Considering the advantage of dimension reduction, automatic decoupling, loop pairing and handling non-square in PLS latent variables space, Kaspar and Ray proposed dynamic PLS (DyPLS) framework for control system design by utilizing the PLS loading matrices to construct precompensators and postcompensators [15, 16]. Based on their work, Chen [17] proposed another dynamic PLS structure with ARX model to design a multi-loop proportional-integral-derivative controller. In the advanced control system design, lots of researches have been done using PLS to identify the predictive model of MPC [18–21]. Those results demonstrated that the method of integrating PLS with MPC is effective and reasonable. That is because this method has great advantages for control system design comparing to traditional MPC schemes. Firstly, it has strong ability to reduce system order, which is very significant for industrial production equipment with hundreds of orders [22]. Secondly, it has strong ability to decouple the system. By this method, the original multiple inputs multiple outputs (MIMO) control system is transformed into multiple single-loop control, which dramatically simplifies the controller design. Thirdly, it has strong ability to eliminate noise, since the residuals caused noise can be ignored in PLS modeling procedure. Fourthly, it has strong ability to

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handle non-square system. Since latent variables are selected in pairs, the number of input and output latent variables is equal. However, the problem of handling constraints commonly exists in this PLS framework. When projecting to latent space, the constraints that were decoupled in original space become coupled. Laskminaryanan reduced the permitted region of original space in order to make the constraints decoupled in latent variable space. However, some set points cannot be reached and some disturbances will not be rejected completely when the implement MPC used their method [23].

In this paper, we propose to apply iterative method to deal with this kind of coupling constraints in latent variable space without changing the permitted regions of original variables. Combining with the advantage of dimension reduction and decoupling in PLS framework, we design MPC scheme in latent subspaces. At last, we will apply this method to a non-square system and slurry reactor, and then compare the control performance with adaptive internal model control proposed by Hu [24] to illustrate the superiority of this method.

2 PLS MODELING FOR CONTROLLER DESIGN

2.1 Partial least squares

Partial least squares is a popular linear regression method based on projecting the information in a high-dimensional space down into a low-dimensional space defined by some latent variables [25].

In standard PLS, the given process data are subdivided into two blocks, a dependent block, input block $X = (x_{ij})_{l \times m}$ and an output block $Y = (y_{ij})_{l \times n}$, where l , m and n denote the numbers of observations, manipulated variables and controlled variables, respectively. The input X and the output Y can be expressed by iteratively subtracting principal components as following outer model:

$$\begin{aligned} X - E^* &= TP^T = \sum_{r=1}^R t_r p_r^T \\ Y - F^* &= UQ^T = \sum_{r=1}^R u_r q_r^T \end{aligned} \quad (1)$$

where R [$R < \max(m, n)$] is the selected number of principal components, which is determined by some heuristic and other statistical methods [26], p_r and q_r are the r th loading vectors of loading matrices P and Q , respectively; t_r and u_r are the r th scores vectors of scores matrices T and U , respectively; and E^* and F^* are residual matrices of X and Y , respectively.

The algebraic relationship between input variables t_r and output variable u_r in the latent space comprise the inner model of PLS.

$$Y - F^* = TBQ^T = \sum_{r=1}^R b_r t_r q_r^T$$

$$\begin{aligned} b_r &= u_r^T t_r / t_r^T t_r \\ u_r &= b_r t_r \end{aligned} \quad (2)$$

where b_r is the regression coefficient of the score model, which comprise matrix B . However, the standard PLS model only deal with static relationship rather than dynamic relationships because of its linear algebraic structure. Thus, the dynamic PLS model method is necessarily introduced to account for the dynamical relationship of processes, exactly.

2.2 Dynamic ARX-PLS model

Many methods have been proposed to incorporated process linear or nonlinear dynamic characters into PLS modeling approach by modifying the traditional PLS model. Qin [27] and Ricker [28] applied multivariable autoregressive moving average (ARMX) and finite impulse response (FIR) model of process variables to PLS at the cost of increasing the dimensions of input and output matrixes, respectively. Kasper and Ray proposed another schemes by inserting dynamic filter before the algebraic PLS model. Another method is integrating autoregressive exogenous (ARX) with PLS. In this paper, linear ARX model is applied into inner model of PLS to represent the dynamic character of process, named autoregressive exogenous-PLS (ARX-PLS). The ARX-PLS inner model can be expressed as:

$$\begin{aligned} u_r &= H_r(t_r) \\ Y - F &= H(T)Q^T = \sum_{r=1}^R H_r q_r^T \end{aligned} \quad (3)$$

and

$$H(T) = \text{diag}[H_1(t_1), \dots, H_R(t_R)]$$

where $H_r(t_r)$ is the linear ARX model formulation of the r th SISO subsystem in latent variable space and F is the residual matrix of Y using ARX-PLS model.

Through the above ARX-PLS modeling method, the MIMO system can be decomposed into several SISO subsystems and the dimension of the MIMO can be reduced to be the total number of SISO subsystem. Then, the controller design of multivariable system can be decoupled with the controller design of several independent single-variable systems.

2.3 Identification of ARX-PLS model using least squares (LS)

In previous subsection, the ARX model of latent variable into PLS for dynamic modeling has been introduced. Considering the identification of ARX-PLS in Eq. (3), a nonlinear optimization model is proposed as follow:

$$\min J_o = \frac{1}{2} \sum_{k=1}^n [y(k) - y^{\text{PLS}}(k)]^T [y(k) - y^{\text{PLS}}(k)] \quad (4)$$

where $y(k)$ and $y^{\text{PLS}}(k)$ denote the process and the ARX-PLS model output vector at time k , respectively. According to orthogonal property of latent spaces, optimization model, Eq. (4) can be transformed into latent spaces as follows:

$$\min J = \frac{1}{2} \sum_{k=1}^n [\mathbf{u}(k) - \mathbf{u}^{\text{PLS}}(k)]^T [\mathbf{u}(k) - \mathbf{u}^{\text{PLS}}(k)] \quad (5)$$

where $\mathbf{u}(k)$ and $\mathbf{u}^{\text{PLS}}(k)$ are the process and ARX-PLS model output vector at time k in latent spaces, respectively. And

$$\mathbf{u}(k) = [u_1(k), u_2(k), \dots, u_R(k)]$$

$$\mathbf{u}^{\text{PLS}}(k) = [u_1^{\text{PLS}}(k), u_2^{\text{PLS}}(k), \dots, u_R^{\text{PLS}}(k)]$$

Then,

$$\begin{aligned} \min J &= \frac{1}{2} \sum_{k=1}^n [\mathbf{u}(k) - \mathbf{u}^{\text{PLS}}(k)]^T [\mathbf{u}(k) - \mathbf{u}^{\text{PLS}}(k)] \\ &= \frac{1}{2} \sum_{r=1}^R \sum_{k=1}^n [u_r(k) - u_r^{\text{PLS}}(k)]^2 \\ &= \min \widetilde{J}_1 + \min \widetilde{J}_2 + \dots + \min \widetilde{J}_r + \min \widetilde{J}_R \quad (6) \end{aligned}$$

where $\widetilde{J}_r = \frac{1}{2} \sum_{k=1}^n [u_r(k) - u_r^{\text{PLS}}(k)]^2$. J_0 , J denote the objective function in the original and latent space, respectively. $\widetilde{J}_r$ denotes the objective function in the r latent space.

The ARX-PLS model output of the r th latent output variable $u_r(k)$ can be expressed as:

$$u_r^{\text{PLS}}(k) = \boldsymbol{\varphi}_r(k) \boldsymbol{\theta}_r(k) \quad (7)$$

$$\begin{aligned} \boldsymbol{\varphi}_r(k) &= [u_r(k-1), u_r(k-2), \dots, u_r(k-n), \\ &\quad t_r(k-d_r-1), t_r(k-d_r-2), \dots, t_r(k-d_r-m)] \end{aligned}$$

Above equation contains the current and past projection information of the r th latent input variables and output variables, where m and n are the model order in latent variable space, respectively; d_r is the delay time of the r th subsystem in latent space. $\boldsymbol{\theta}_r(k) = [-a_{r1}(k), -a_{r2}(k), \dots, -a_{rn}(k), -b_{r1}(k), -b_{r2}(k), \dots, -b_{rn}(k)]^T$ is ARX model parameter vector estimated at time k and can be obtained by recursive least squares:

$$\boldsymbol{\theta}_r(k) = [\boldsymbol{\varphi}_r^T(k) \boldsymbol{\varphi}_r(k)]^{-1} \boldsymbol{\varphi}_r^T(k) \mathbf{u}_r(k) \quad (8)$$

Then, the optimization question under PLS model frame can be rewritten as:

$$\begin{aligned} \min \widetilde{J}_r &= \frac{1}{2} \sum_{k=1}^n [u_r(k) - u_r^{\text{PLS}}(k)]^2 \\ &= \frac{1}{2} \sum_{k=1}^n [u_r(k) - \boldsymbol{\varphi}_r(k) \boldsymbol{\theta}_r(k)]^2 \quad (9) \end{aligned}$$

Note that the whole procedure is done by iterative

method off-line; so when new input and output data became available, the identification processes must be restarted. And parameter m , n and d_r should be determined before the parameters of ARX-PLS model are iteratively estimated.

While the above identification method only considers the identification errors of inner model, it is needed to verify whether the errors of the model identification in original spaces E has exceeded the threshold ε by projecting the inner model outputs to the original control variables outputs.

$$E = \sum_{k=1}^n [\mathbf{y}(k) - \mathbf{y}^{\text{PLS}}(k)]^T [\mathbf{y}(k) - \mathbf{y}^{\text{PLS}}(k)] \leq \varepsilon$$

Then, replacing $\mathbf{y}^{\text{PLS}}(k)$ with Eqs. (3), (6) and (7), E can be rewritten as:

$$\begin{aligned} E &= \frac{1}{2} \sum_{k=1}^n [\mathbf{y}(k) - \mathbf{u}^{\text{PLS}}(k) \mathbf{Q}^T]^T [\mathbf{y}(k) - \mathbf{u}^{\text{PLS}}(k) \mathbf{Q}^T] \\ &= \sum_{k=1}^n \left[\mathbf{y}(k) - \sum_{r=1}^R u_r^{\text{PLS}}(k) \mathbf{q}_r^T \right]^T \left[\mathbf{y}(k) - \sum_{r=1}^R u_r^{\text{PLS}}(k) \mathbf{q}_r^T \right] \\ &= \sum_{k=1}^n \left[\mathbf{y}(k) - \sum_{r=1}^R \boldsymbol{\varphi}_r(k) \boldsymbol{\theta}_r(k) \mathbf{q}_r^T \right]^T \times \\ &\quad \left[\mathbf{y}(k) - \sum_{r=1}^R \boldsymbol{\varphi}_r(k) \boldsymbol{\theta}_r(k) \mathbf{q}_r^T \right] \leq \varepsilon \end{aligned}$$

Therefore, when the output error exceeds to the threshold ($E > \varepsilon$), the proposed identification procedure is restarted until the output error is smaller than the predefined threshold.

3 CONSTRAINED ITERATIVE MODEL PREDICTIVE CONTROL DESIGN UNDER ARX-PLS FRAMEWORK

Model predictive control is a control scheme in which the current control action is obtained by solving a finite horizon open-loop optimal control problem at each sampling instant. Using the current state of the plant as the initial state; the optimization yields an optimal control sequence and only the first control in this sequence is applied to the plant [8].

3.1 Constrained MPC control scheme

In many control applications, the desired performance cannot be expressed solely as trajectory following problem. Many practical requirements are more naturally expressed as constraints on process variables [29]. Most of interest in model predictive control comes from its ability to handle constraints. In this section, we will describe a standard formulation of model predictive control with the present of constraints is described and a quadratic program to solve the constraint case is introduced.

For SISO system, the control increment of input variable Δu can be calculated by solving the following objection function:

$$\min_{\Delta U} J(k) = \|\omega(k) - \hat{y}_M(k)\|_Q^2 + \|\Delta U(k)\|_R^2 \quad (10)$$

where $\omega(k) = [\omega(k+1), \omega(k+2), \dots, \omega(k+P)]^T$

$\Delta U(k) = [\Delta u(k), \Delta u(k+1), \dots, \Delta u(k+M-1)]^T$

$\hat{y}_M(k) = [\hat{y}_M(k+1|k), \hat{y}_M(k+2|k), \dots, \hat{y}_M(k+P|k)]^T$
($i=1, \dots, P$)

$$A = \begin{bmatrix} a_1 & \dots & 0 \\ \dots & \dots & \dots \\ a_M & \dots & a_1 \\ \dots & \dots & \dots \\ a_P & \dots & a_{P-M+1} \end{bmatrix}$$

$$Q = \text{diag}(q_1, q_2, \dots, q_P)$$

$$R = \text{diag}(r_1, r_2, \dots, r_M)$$

where M is called the control horizon, P is the predictive horizon, a_i ($i=1, \dots, P$) are sampled output values for step response of system. Q and R are the weighting matrices of tracking error and input increment respectively, which indicate the inhibition to tracking error and control action variation, respectively; $\Delta u(k+j-1) = u(k+j-1) - u(k+j-2)$, $j=1, \dots, M$; $\hat{y}_M(k)$ is the model prediction of output at time k when the information update to the current time, $\omega(k+i)$ is the reference trajectory $\omega(k+i) = \alpha\omega(k+i-1) + (1-\alpha)r(k+i)$, α is smooth parameter between 0 and 1, and the reference $r(k+i)$ is the expected output of closed system.

Then, the model prediction of output at future time $k+i$ can be given as

$$\hat{y}_M(k+i|k) = \tilde{y}_0(k+1|k) + \sum_{j=1}^{\min(M,i)} a_{i-j+1} \Delta u(k+j-1),$$

$$i=1, \dots, P; \quad j=1, \dots, M \quad (11)$$

Replacing $\hat{y}_M(k)$ with Eq. (11), the objection function can be rewritten as

$$\min_{\Delta U} J(k) = \|\omega(k) - \tilde{Y}_0(k+1) - A\Delta U(k)\|_Q^2 + \|\Delta U(k)\|_R^2 \quad (12)$$

and

$\tilde{Y}_0(k+1) = [\tilde{y}_0(k+1|k), \tilde{y}_0(k+2|k), \dots, \tilde{y}_0(k+P|k)]^T$;
where $\tilde{y}_0(k+i|k)$ denotes the initial predictive value of $y(k+i)$ at time t .

A real industry process has to be subject to specified bounds due to safety limitation, environmental regulations, consumer specifications and physical restriction such as min or mum temperature, pressure

and level limits in reactor. Taking these limitations as constraints into account with form of linear matrix inequality, the constrained MPC scheme can be summarized as (only the manipulated variables constraints are considered in this paper for simplicity)

$$\min_{\Delta U(k)} J = \|\omega(k) - \tilde{Y}_0(k+1) - A\Delta U(k)\|_Q^2 + \|\Delta U(k)\|_R^2$$

$$\text{s.t.} \quad C\Delta U(k) \leq B \quad (13)$$

where C is coefficient matrix of input variables constraints. It can be transformed into standard quadratic program formation to calculate future input moves $\Delta U(k)$ at every sampling instant as follow:

$$\min_{\Delta U(k)} J = \frac{1}{2} \Delta U(k)^T [2(A^T Q A + R)] \Delta U(k) +$$

$$[-2(\omega(k) - \tilde{Y}_0(k+1|k))^T A] \Delta U(k)$$

$$\text{s.t.} \quad C\Delta U(k) \leq B \quad (14)$$

The m input moves can be obtained by solving the above quadratic program at each sample point and only the first control action $\Delta U(k)$ can be implementing on the plant.

3.2 Constrained MPC controller design in dynamic PLS Framework

In Fig. 1 Y_{sp} denotes set point vector, D denotes disturbance vector, W_X and W_Y are the input and output scaling matrixes, respectively, ARX-PLS-LVMPC denotes the multi-loop MPC controller block based on ARX-PLS model [*i.e.*, Eq. (7) in above section] in latent variables.

In this framework, the constrains on the original input variables X will be discussed. In the control block, (ARX-PLS-LVMPC), the constrained MPC strategy of the r th control loop is designed based on the linear ARX model between u (input) and t (output) latent variables at the instant k in the r th latent variable subspace (R_r is the weighted matrix):

$$\min_{\Delta T(k)} J_r = \frac{1}{2} \Delta T_r(k)^T [2(A_r^T Q_r A_r + R_r)] \Delta T_r(k) +$$

$$[-2(\omega_r(k) - \tilde{U}_{0r}(k+1|k))^T A_r] \Delta T_r(k)$$

$$\text{s.t.} \quad C_r^* \Delta T_r(k) \leq B_r^* \quad (15)$$

The constraints on original variables can be transformed into constraints about latent variables in each latent subspace, in which C_r^* and B_r^* can be obtained by the following transformation:

$$T(k) = P^+ X(k)$$

$$U(k) = Q^+ Y(k) \quad (16)$$

where P^+ and Q^+ are the generalized inverse of P and Q , respectively.

Consider a $m \times n$ system that is modeled using

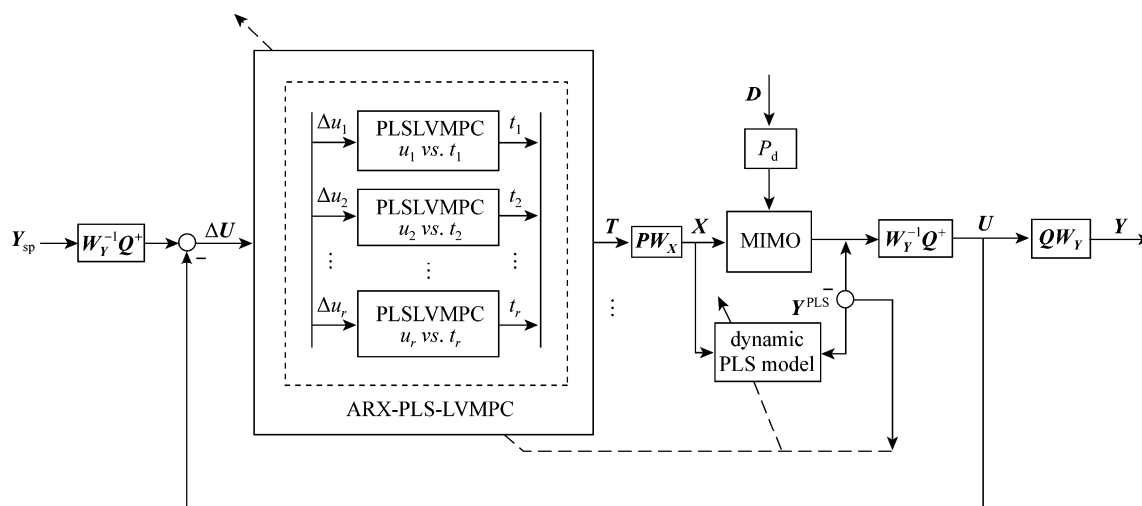


Figure 1 Multi-loop MPC based on dynamic PLS model

ARX-PLS model by choosing f [$f \leq \max(m, n)$] principal components. The constraints on input variables in original space are irrelevant with each other and can be expressed by the inequality:

$$X_{\min} \leq X(k) \leq X_{\max} \quad (17)$$

Replacing $X(k)$ with Eq. (1), then the constraints projected to latent spaces is obtained as follow:

$$X_{\min} \leq T(k)P^T \leq X_{\max}$$

$$\begin{bmatrix} X_{1\min} \\ X_{2\min} \\ \vdots \\ X_{f\min} \end{bmatrix} \leq \begin{bmatrix} p_{11}t_1(k) + p_{12}t_2(k) + \dots + p_{1f}t_f(k) \\ p_{21}t_1(k) + p_{22}t_2(k) + \dots + p_{2f}t_f(k) \\ \vdots \\ p_{f1}t_1(k) + p_{f2}t_2(k) + \dots + p_{ff}t_f(k) \end{bmatrix} \leq \begin{bmatrix} X_{1\max} \\ X_{2\max} \\ \vdots \\ X_{f\max} \end{bmatrix} \quad (18)$$

The constraints on latent input variables can be obtained by solving the above inequality, but they are coupled. Therefore, an iterative method will be introduced to cope with the coupling constraints for multi-loop MPC controllers' design of latent variables. For the first latent variable $t_1(k)$, the constraints on $t_1(k)$ can be expressed as follow:

$$\begin{bmatrix} P_1^* \\ -P_1^* \end{bmatrix} t_1(k) \leq \begin{bmatrix} X_{\max} - F[t_2(k), \dots, t_f(k)] \\ -X_{\min} + F[t_2(k), \dots, t_f(k)] \end{bmatrix}$$

where $P_1^* = [p_{11}, p_{21}, \dots, p_{f1}]^T$, p_{ij} is the element of the loading matrices P . F denotes linear function of input variables in latent space at time k . For MPC controller design, due to $t_1(k) = t_1(k-1) + \Delta t_1(k)$, the inequality constraints on input moves can be rewritten as:

$$\begin{bmatrix} P_1^* \\ -P_1^* \end{bmatrix} \Delta t_1(k) \leq \begin{bmatrix} X_{\max} - F[t_1(k-1), t_2(k), \dots, t_f(k)] \\ -X_{\min} + F[t_1(k-1), t_2(k), \dots, t_f(k)] \end{bmatrix} \quad (19)$$

Before solution procedure of control law is started at instant k , latent variable $t_i(k)$ ($i=1, 2, \dots, f$) cannot be changed at current time and can be replaced by the value at last time $t_i(k-1)$ ($i=1, 2, \dots, f$), so the inequality constraints on input moves $\Delta t_1(k)$ can be replaced by:

$$\begin{bmatrix} P_1^* \\ -P_1^* \end{bmatrix} \Delta t_1(k) \leq \begin{bmatrix} X_{\max} - F[t_1(k-1), t_2(k-1), \dots, t_f(k-1)] \\ -X_{\min} + F[t_1(k-1), t_2(k-1), \dots, t_f(k-1)] \end{bmatrix} \quad (20)$$

When turn to start the MPC strategy of the r th latent variable $t_r(k)$, the controller design processes of $r-1$ latent variables have finished, then $t_i(k)$ ($i=1, \dots, r-1$) has been obtained by solving control moves $\Delta t_i(k)$. The inequality constraints on the r th input moves $\Delta t_r(k)$ can be given by:

$$\begin{bmatrix} P_r^* \\ -P_r^* \end{bmatrix} \Delta t_r(k) \leq \begin{bmatrix} X_{\max} - F[t_1(k), \dots, t_{r-1}(k), t_r(k-1), \dots, t_f(k-1)] \\ -X_{\min} + F[t_1(k), \dots, t_{r-1}(k), t_r(k-1), \dots, t_f(k-1)] \end{bmatrix} \quad (21)$$

By applying this iterative method, the r th control loop has been decoupled at the each sample point.

3.3 Algorithm for constraint iterative MPC in ARX-PLS framework

The algorithm for constraint iterative MPC is

Table 1 Constraint iterative MPC algorithm

Algorithm: constraint iterative MPC

Require: Identification of ARX-PLS model. Train the ARX-PLS model using observational data, until the output error is permitted ($E < \varepsilon$).

Multi-loop MPC controller design:

- 1 Initiate parameters of MPC controller, $\mathbf{Q}, \mathbf{R}, h_r, \alpha_r$ initiate $k = 1, r = 1$
- 2 While $k < N$ do
 - While $r < R$ do
 - Calculate the step response series a_r of subsystem and construct matrix $\mathbf{A}_r$
 - Calculate constraints of the r th latent variables by Eq. (24)
 - Start SISO MPC methods for the r th latent variable
 - Calculate optimal moves $\Delta t_r(k)$ at the current time k by solving $\mathbf{QP}$ Eq. (15)
 - $r = r + 1$
 - $k = k + 1$
- 3 Turing the parameters of ARX-PLS-LVMPC, go back to step 2, until obtain expected performance

shown in Table 1.

Though the above algorithm only considers the manipulated variable constraints, the manipulated variable rate constraints and output variable constraints can also be transformed into the constraints on the input variable in latent variable space. The output variable constraints can be illustrated by:

$$\mathbf{Y}_{\min} \leq \mathbf{Y}(k) \leq \mathbf{Y}_{\max}$$

$$\mathbf{Y}_{\min} \leq \mathbf{U}(k)\mathbf{Q}^T \leq \mathbf{Y}_{\max}$$

Replaced by Eq. (3)

$$\mathbf{Y}_{\min} \leq \mathbf{H}[\mathbf{T}(k)]\mathbf{Q}^T \leq \mathbf{Y}_{\max}$$

Similar to the condition of Eq. (22), the constraints on output variables can be included in the optimization step along with the input constraints for each control loop.

4 EXAMPLES

To illustrate the advantage of the constraint iterative MPC scheme based on ARX-PLS framework; two study cases are discussed separately as follow.

4.1 Study Case 1: non-square system

Non-square systems with unequal numbers of inputs and outputs are commonly encountered in the industry processes. In control process appropriate number of inputs and outputs is added or deleted to make system square and gain convenience of control design [30]. A system with four inputs and three outputs proposed by Kalman [31] is chosen to illustrate the control performance of the proposed algorithm. The transfer function of Kalman system $G(s)$ is given as:

$$G(s) = \begin{bmatrix} \frac{3(s+3)(s+5)}{(s+1)(s+2)(s+5)} & \frac{6(s+1)}{(s+2)(s+4)} & \frac{2s+7}{(s+3)(s+4)} & \frac{2s+5}{(s+2)(s+3)} \\ \frac{2}{(s+3)(s+5)} & \frac{1}{s+1} & \frac{2s+7}{(s+1)(s+2)(s+3)} & \frac{8(s+2)}{(s+1)(s+2)(s+3)} \\ \frac{2(s^2+7s+18)}{(s+1)(s+2)(s+3)} & \frac{-2s}{(s+1)(s+3)} & \frac{1}{s+3} & \frac{2(5s^2+27s+34)}{(s+1)(s+3)(s+5)} \end{bmatrix}$$

In order to build up the ARX-PLS model, the identification data set is chosen with 1000 samples

obtained, and the resulted of ARX-PLS model are derived by choosing four principal components as follows:

$$\mathbf{Q}^T \mathbf{W}_Y = \begin{bmatrix} 0.7077 & 0.3399 & 0.7166 \\ 1.2263 & -0.2445 & 0.2430 \\ -0.2207 & -0.2273 & 1.0748 \\ -0.2243 & 0.2103 & 1.0911 \end{bmatrix}, \quad \mathbf{W}_X \mathbf{P} = \begin{bmatrix} 0.0995 & 0.2020 & -0.0362 & 0.0041 \\ 0.1209 & -0.0690 & -0.0439 & -0.1834 \\ 0.1062 & -0.0663 & -0.1241 & 0.1338 \\ 0.1338 & -0.0657 & 0.1828 & 0.0067 \end{bmatrix}$$

$$H(z) = \begin{bmatrix} \frac{0.7754z - 0.4100}{z^2 - 0.7071z + 0.0267} & & & \\ & \frac{0.3965z - 0.1444}{z^2 - 0.8455z + 0.1562} & & \\ & & \frac{0.4704z - 0.3287}{z^2 - 0.9120z + 0.1143} & \\ & & & \frac{0.0958z - 0.0142}{z^2 - 0.3810z - 0.0301} \end{bmatrix}$$

About tuning parameters available in proposed MPC algorithm for each latent variable, the procedure comprised, adjustments on the weighting diagonal matrixes Q_r and R_r , trajectory reference parameter α_r , feedback parameter h_r , when control and prediction horizon were selected (2 and 6, respectively). First tuning parameters set was selected (Table 2) for entire run.

Table 2 MPC tuning parameters for four control loops in latent variable space

Principal component	Parameters						
	P	M	Q	R	α	h	β
1	6	2	0.5	1	0.5	1	0.2
2	6	2	0.5	1	0.5	1	0.2
3	6	2	0.5	1	0.5	1	0.2
4	6	2	0.5	1	0.5	1	0.2

M denotes control time domain constraint region

$$H(z) = \begin{bmatrix} \frac{0.8410z - 0.2916}{z^2 - 0.5442z - 0.0041} & & & \\ & \frac{0.3519z - 0.1633}{z^2 - 0.9793z + 0.2170} & & \\ & & \frac{0.4336z - 0.3806}{z^2 - 1.2403z + 0.3174} & \end{bmatrix}$$

First tuning parameters set was selected (Table 3) for entire run.

Table 3 MPC tuning parameters for three control loops in latent variable space

Principal component	Parameters						
	P	M	Q	R	α	h	β
1	6	2	0.5	1	0.1	1	0.2
2	6	2	0.5	1	0.1	1	0.2
3	6	2	0.5	1	0.1	1	0.2

The constraint region of x_1 , x_2 , x_3 and x_4 are $[-0.5$,

of x_1 is $[-0.5, 0.5]$; constraints region of x_2 is $[-0.4, 0.4]$; and constraints region of x_3 is $[-1, 1]$; constraints region of x_4 is $[-1, 1]$, Fig. 2 presents the simulation results for outputs considered using the parameters of Table 2.

To illustrate the dimension reduction feature of PLS, the resulting of ARX-PLS model parameters were derived by choosing three principal components as follows:

$$Q^T W_Y = \begin{bmatrix} 0.5762 & 0.2792 & 0.6713 \\ 1.0464 & -0.2116 & 0.1384 \\ -0.1887 & -0.2810 & 0.8252 \end{bmatrix}$$

$$W_X P = \begin{bmatrix} 0.0808 & 0.1576 & -0.0077 \\ 0.0749 & 0.0286 & -0.2004 \\ 0.1071 & -0.1182 & -0.0263 \\ 0.1381 & -0.0465 & 0.1008 \end{bmatrix}$$

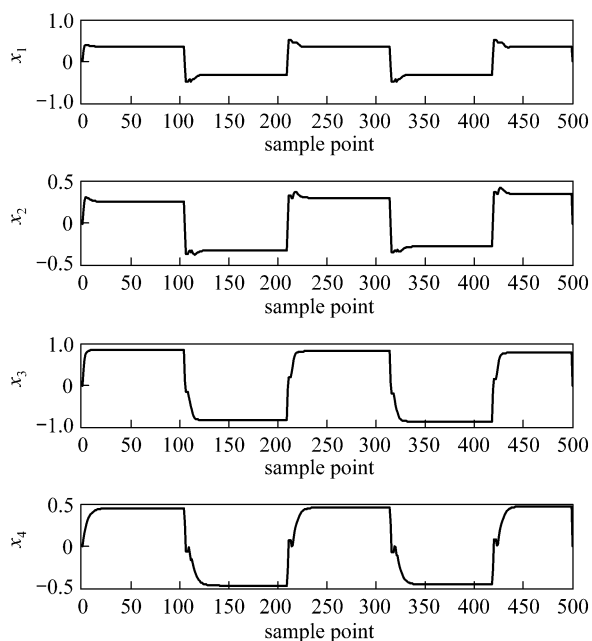
0.5], $[-1, 1]$, $[-1, 1]$ and $[-1, 1]$, respectively. Fig. 3 presents the simulation results for outputs considered using the parameters of Table 3.

When two principle components are chosen, the ARX-PLS model parameter is as follow:

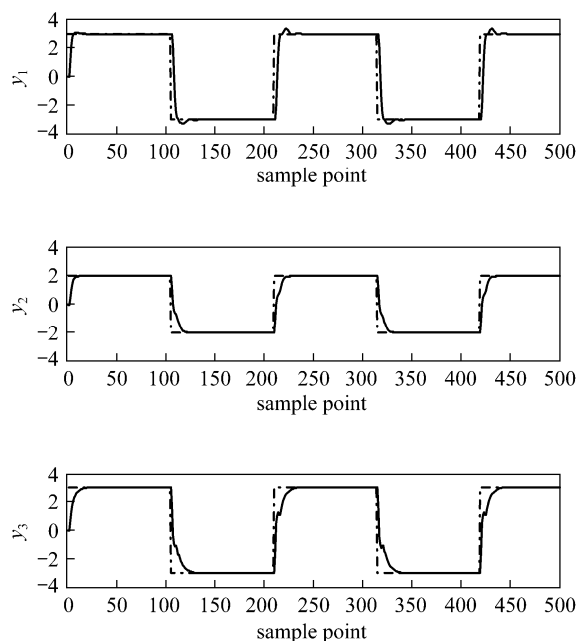
$$Q^T W_Y = \begin{bmatrix} 0.7027 & 0.2019 & 0.7432 \\ -0.3224 & 0.4167 & -0.3330 \end{bmatrix}$$

$$W_X P = \begin{bmatrix} 0.0264 & 0.0908 \\ 0.0836 & 0.1845 \\ 0.1483 & -0.0261 \end{bmatrix}$$

$$H(z) = \begin{bmatrix} \frac{0.8854z - 0.0615}{z^2 - 0.3679z - 0.0057} \\ \frac{0.3790z - 0.0932}{z^2 - 0.5626z + 0.0863} \end{bmatrix}$$



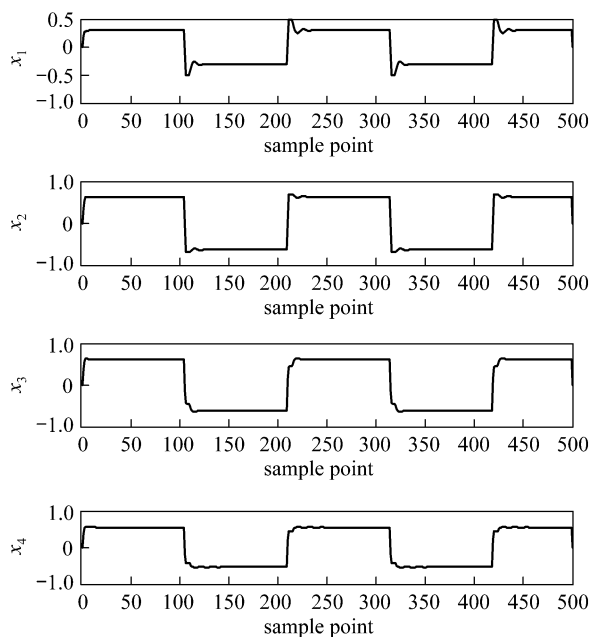
(a) Constraint input simulation



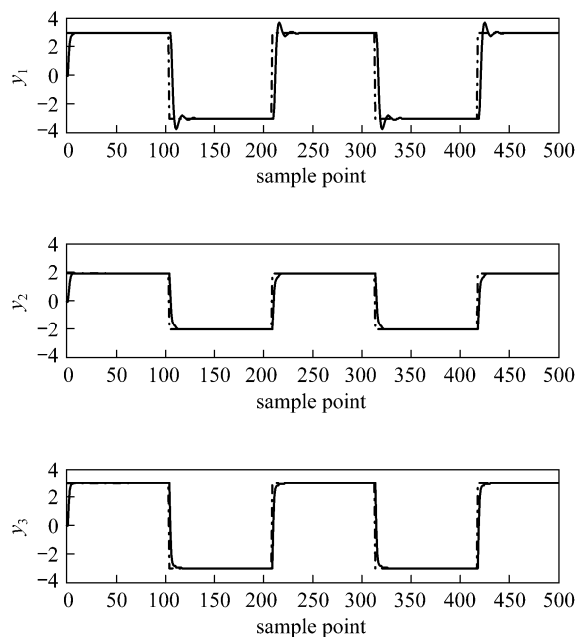
(b) Output response of control system by choosing four principal component

Figure 2 Response of system after turning the MPC parameters in ARX-PLS framework

----- set points; — output of MPC controller



(a) Constraint input simulation



(b) Output response of control system by choosing three principal component

Figure 3 Response of system after turning the MPC parameters in ARX-PLS framework

----- set points; — output of MPC controller

First tuning parameters set is selected (Table 4) for entire run.

Table 4 MPC tuning parameters for two control loops in latent variable space

Principal component	Parameters						
	P	M	Q	R	α	h	β
1	6	2	0.5	1	0.5	1	0.2
2	6	2	0.5	1	0.5	1	0.2

The constraint region of x_1 , x_2 , x_3 and x_4 are $[-0.5, 0.5]$, $[-0.4, 0.4]$, $[-1, 1]$ and $[-1, 1]$, respectively. Fig. 4 presents the simulation results for outputs considered using the parameters of Table 4.

The above result has shown that the steady-state error (normally defined: $y_{SSE} = [y(\infty) - y_{sp}] / y(\infty)$) exists in output result of the system. Because the system cannot be illustrated exactly when the modeling process chooses two principal components, but the max steady-state error of those three output is about 4% and less than 5% [32].

4.2 Study Case 2: a slurry reactor for ethylene polymerizations

The second example is considered mainly based on the following two aspects. First, polyethylene is the largest synthetic polymer in terms of production and stirred-tank reactor for ethylene polymerizations is a significant part. Thus, it is necessary to research the

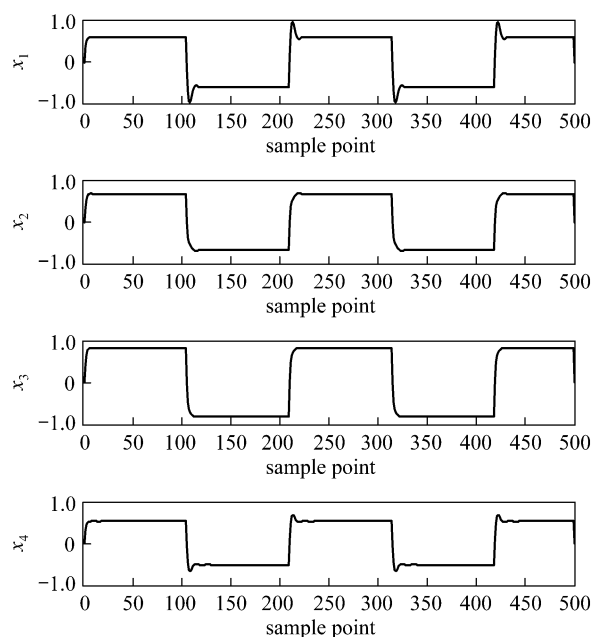
control schemes of reactor, especially it is essential to study the control schemes of the melt index and density. Second, the internal model control (IMC) is widely used in industry process and proved to be more efficient than other traditional method such as proportion integration differentiation control (PID) [33]. Therefore, the process relative to melt index and density is chosen as a study case in Fontes' reactor [34]. And IMC algorithm is used as a comparison with its application to this non-square system.

Consider the 4×2 subsystem in Fontes' reactors with the discrete function as follow:

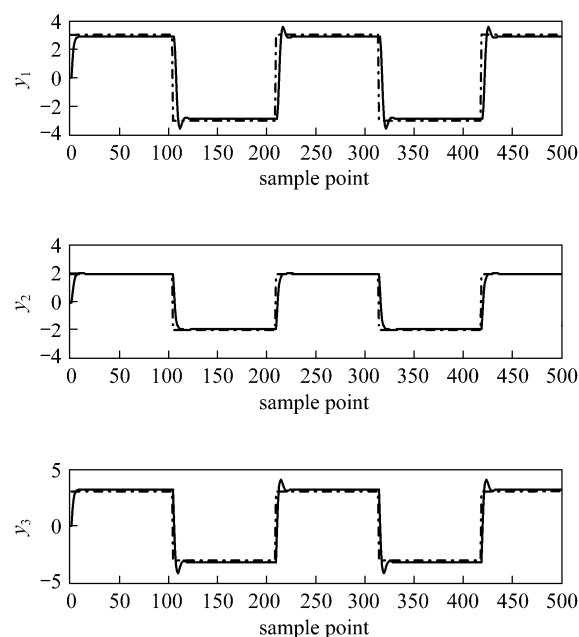
$$\begin{aligned} (1 - 0.4539q^{-1}) \cdot MI(t) &= 0.0417q^{-1} \cdot R_{am}(t) + \\ &0.7716q^{-3} \cdot V_{cat}(t) + 0.1571q^{-5} \cdot R_{ccc}(t) - \\ &0.2054q^{-1} \cdot R_{cmm}(t) + 0.1899q^{-8} \cdot R_{ads}(t) \\ (1 - 0.5375q^{-1} + 0.14q^{-2} - 0.2849q^{-3}) \cdot \rho_s(t) &= \\ &0.9514 \times 10^{-3} \cdot q^{-3} \cdot R_{am}(t) - \\ &0.7625 \times 10^{-3} \cdot q^{-2} \cdot R_{cmm}(t) + 0.48 \times 10^{-3} \cdot q^{-3} \cdot R_{ads}(t) \end{aligned}$$

This system is a typically non-square system presenting four input variables and two output variables as can be viewed in Table 5.

To build the ARX-PLS Model, four input pseudorandom signals with magnitudes ranging from 0.2 to 1 are applied to excite the system and the system responses are collected as a part of the modeling data. The ARX-PLS model parameters are derived by choosing four principal components as follows:



(a) Constraint input simulation



(b) Output response of control system by choosing two principal component

Figure 4 Response of system after turning the MPC parameters in ARX-PLS framework

----- set points; — output of MPC controller

Table 5 Input (manipulated) variables and output variables considered in closed simulation

Input variables	Output variables	Disturbance
hydrogen/ethylene ratio (R_{am})	polymer melt-index (MI)	catalyst feed flow (V_{cat})
cocatalyst/catalyst feed ratio (R_{ccc})	polymer density (ρ_s)	
comonomer/monomer feed ratio (R_{cmm})		
additive/solvent feed ratio (R_{ads})		

$$Q^T W_Y = \begin{bmatrix} 0.1043 & 0.0006 \\ -0.0545 & 0.0008 \\ 0.0708 & 0.0007 \\ 0.0756 & -0.0007 \end{bmatrix}, \quad W_X P = \begin{bmatrix} 0.0773 & 0.2117 & -0.0870 & -0.0163 \\ 0.1139 & -0.0794 & -0.1634 & -0.0177 \\ -0.1677 & 0.0653 & -0.1173 & 0.1236 \\ 0.0992 & -0.0070 & 0.0726 & 0.1995 \end{bmatrix}$$

$$H(z) = \begin{bmatrix} \frac{0.1319z + 0.0050}{z^2 - 1.2657z + 0.3735} z^{-1} & \frac{-0.0492z + 0.1702}{z^2 - 0.8478z + 0.0512} z^{-1} & \frac{0.1114z - 0.0972}{z^2 - 1.1416z + 0.2550} z^{-1} & \frac{0.0453z - 0.0262}{z^2 - 0.9856z + 0.2031} z^{-1} \end{bmatrix}$$

$$\theta = \begin{bmatrix} -1.2657 & -0.8478 & -1.1416 & -0.9856 \\ 0.3735 & 0.0512 & 0.2550 & 0.2031 \\ 0.1319 & -0.0492 & 0.1114 & 0.0453 \\ 0.0050 & 0.1702 & -0.0972 & -0.0262 \end{bmatrix}$$

Where $H(z)$ denotes the ARX-PLS model; and θ is the ARX model parameter. First tuning parameters set is selected (Table 6) for entire run.

Table 6 MPC tuning parameters for four control loops in latent variable space

Principal component	Parameters						
	P	M	Q	R	α	h	β
1	10	2	0.5	1	0.1	1	0.1
2	10	2	0.5	1	0.1	1	0.1
3	10	2	0.5	1	0.8	1	0.1
4	10	2	1	1	0.2	1	0.1

Constraint region of R_{am} , R_{ccc} , R_{cmm} and R_{ads} is $[-0.5, 0.5]$, $[-0.5, 0.5]$, $[-0.5, 0.5]$ and $[-0.5, 0.1]$, respectively, and the disturbance V_{cat} is a step single with magnitude 0.01. Fig. 5 presents the simulation results for outputs considered using the parameters of Table 6.

To demonstrate the dimension reduction feature of PLS, the ARX-PLS model parameters are derived by choosing three principal components as follow:

$$Q^T W_Y = \begin{bmatrix} 0.1031 & 0.0006 \\ 0.1167 & 0.0005 \\ 0.1066 & -0.0006 \end{bmatrix}, \quad W_X P = \begin{bmatrix} 0.1307 & -0.0646 & -0.1117 \\ 0.1374 & -0.0629 & 0.1946 \\ -0.0084 & -0.2009 & -0.0109 \\ 0.1387 & -0.0190 & -0.0589 \end{bmatrix}$$

$$H(z) = \begin{bmatrix} \frac{0.0255z + 0.0553}{z^2 - 1.3622z + 0.4406} z^{-1} & \frac{0.1610z - 0.1377}{z^2 - 1.2148z + 0.2938} z^{-1} & \frac{-0.0564z + 0.1429}{z^2 - 0.9821z + 0.1593} z^{-1} \end{bmatrix}$$

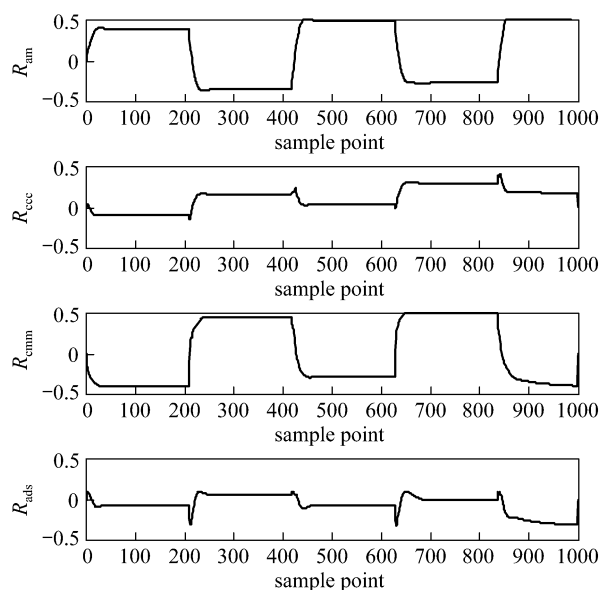
$$\theta = \begin{bmatrix} -1.3622 & -1.2148 & -0.9821 \\ 0.4406 & 0.2938 & 0.1593 \\ 0.0255 & 0.1610 & -0.0564 \\ 0.0553 & -0.1377 & 0.1429 \end{bmatrix}$$

Similarly, first tuning parameters set is selected (Table 7) for entire run.

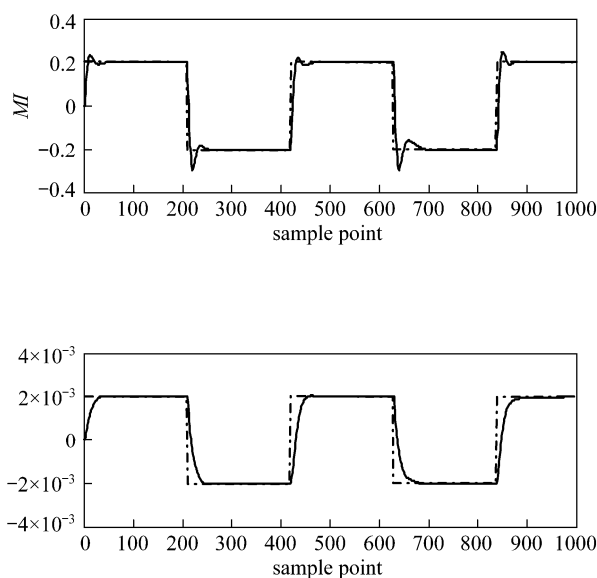
The constraint region of R_{am} , R_{ccc} , R_{cmm} and R_{ads} is $[-0.5, 0.5]$, $[-0.5, 0.5]$, $[-0.5, 0.5]$ and $[-0.5, 0.1]$, respectively, and the disturbance V_{cat} is a step single with magnitude 0.01. Fig. 6 presents the simulation results of outputs in dimensionless condition using the parameters of Table 7.

Table 7 MPC tuning parameters for three control loops in latent variable space

Principal component	Parameters						
	P	M	Q	R	α	h	β
1	10	2	0.5	1	0.1	1	0.1
2	10	2	0.5	1	0.1	1	0.1
3	10	2	0.5	1	0.8	1	0.1



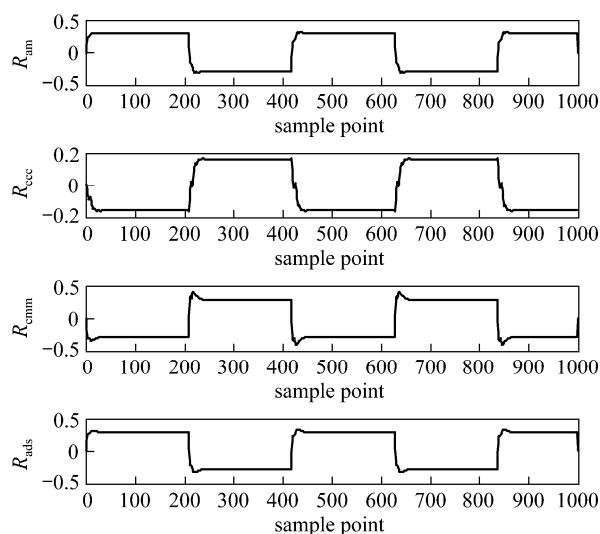
(a) Constraint input simulation



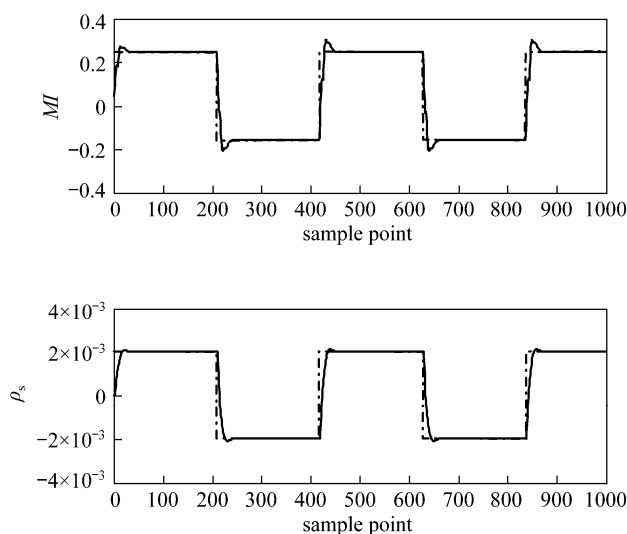
(b) Output response of control system by choosing four principal component

Figure 5 Response of system after turning the MPC parameters in ARX-PLS framework

----- set points; — output of MPC controller



(a) Constraint input simulation



(b) Output response of control system by choosing three principal component

Figure 6 Response of system after turning the MPC parameters in dimension reduction ARX-PLS framework

----- set points; — output of MPC controller

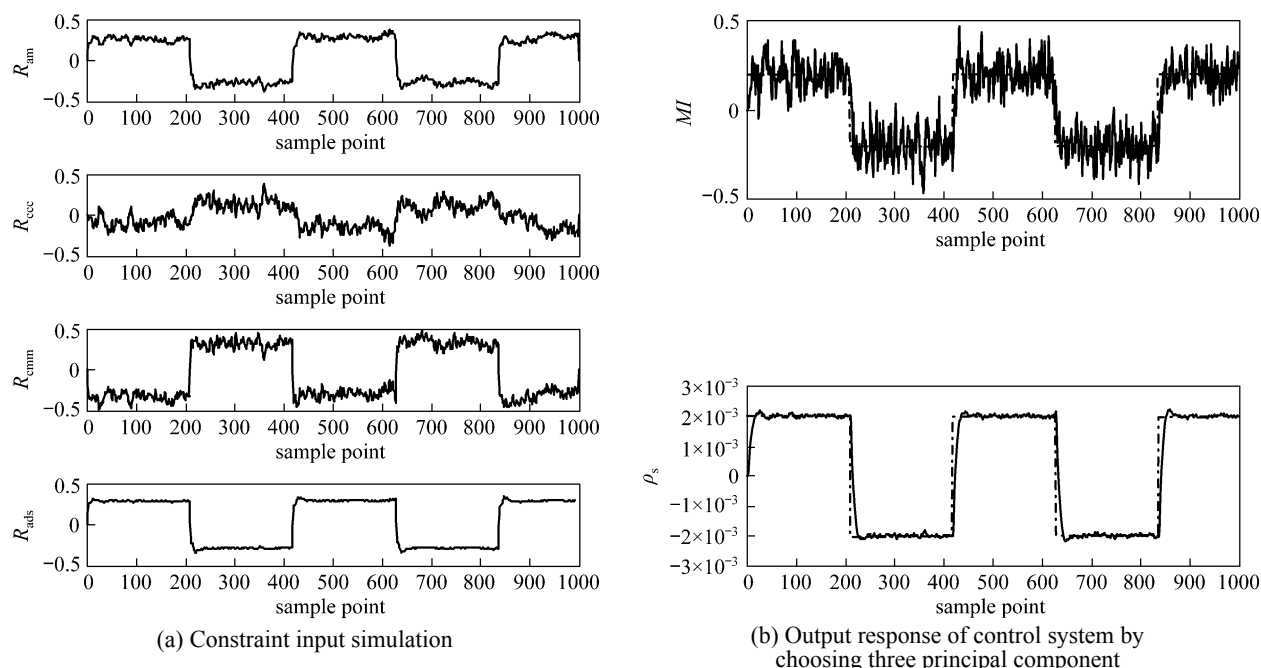


Figure 7 Response of system adding zero-mean Gaussian white noise disturbance in dimensionless ARX-PLS framework
 - - - - set points; — output of MPC controller

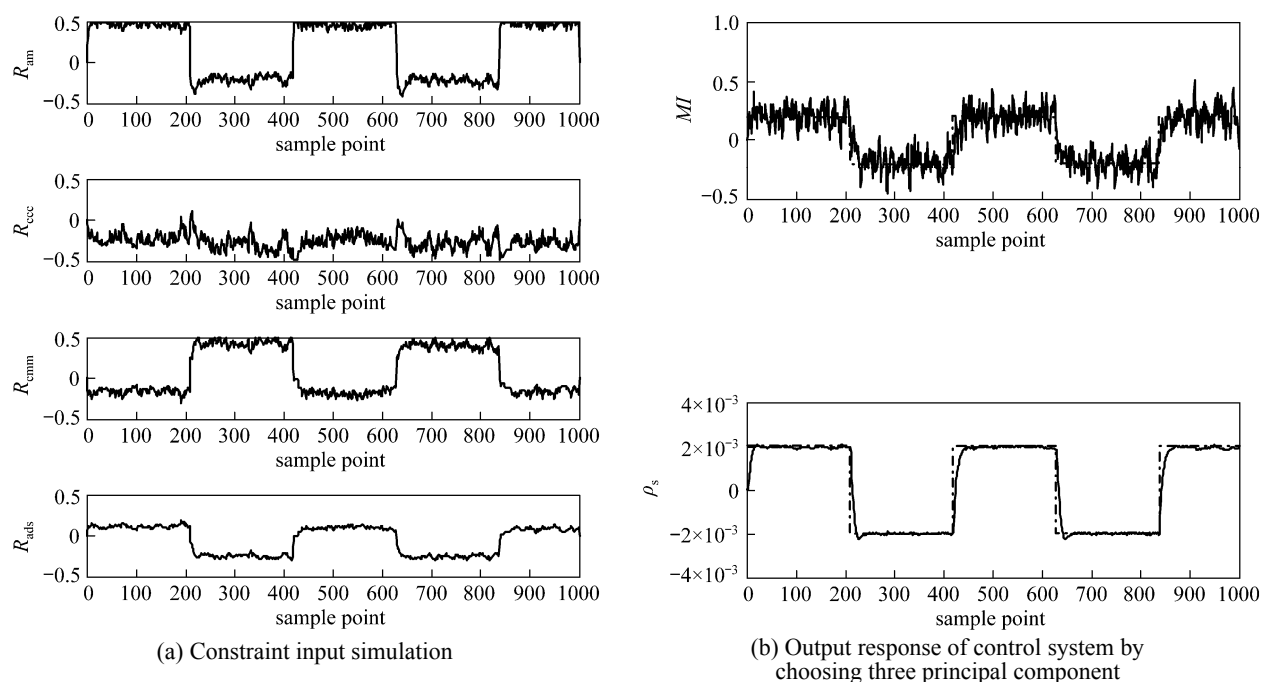


Figure 8 Response of system adding non-zero mean Gaussian white noise disturbance in dimensionless ARX-PLS framework
 - - - - set points; — output of MPC controller

Through choosing optimal modeling parameters and control parameters, the algorithm in dimensionless condition can obtain a good tracking control results as the full dimension condition (Figs. 5 and 6). With the disturbance replaced by zero-mean Gaussian white noise with deviation 0.01 and non-zero mean Gaussian white noise, the control performance cannot

be satisfied by tuning parameters (Figs. 7 and 8). Thus, the robustness of this method will be considered in future work.

Internal model control is applied to this study case in ARX-PLS Framework. The simulations results can be shown in Figs. 9 and 10. By comparing the response of system using MPC controller and using

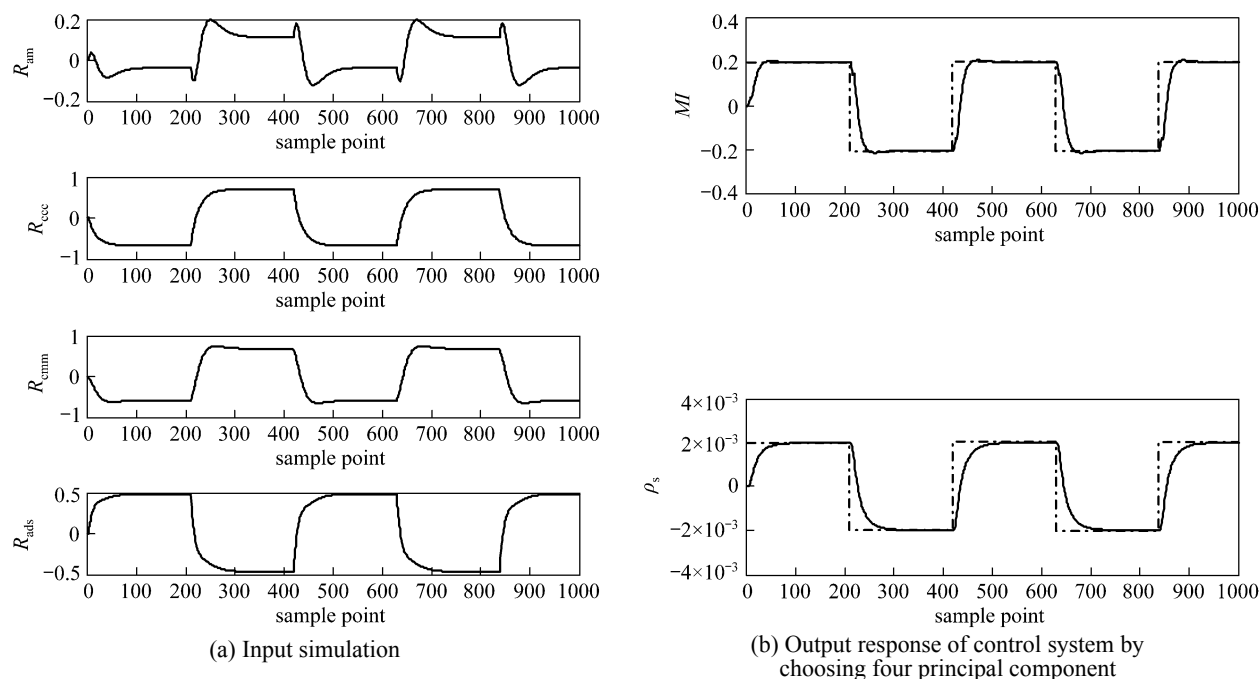


Figure 9 Response of system using IMC scheme in ARX-PLS framework

----- set points; — output of IMC controller

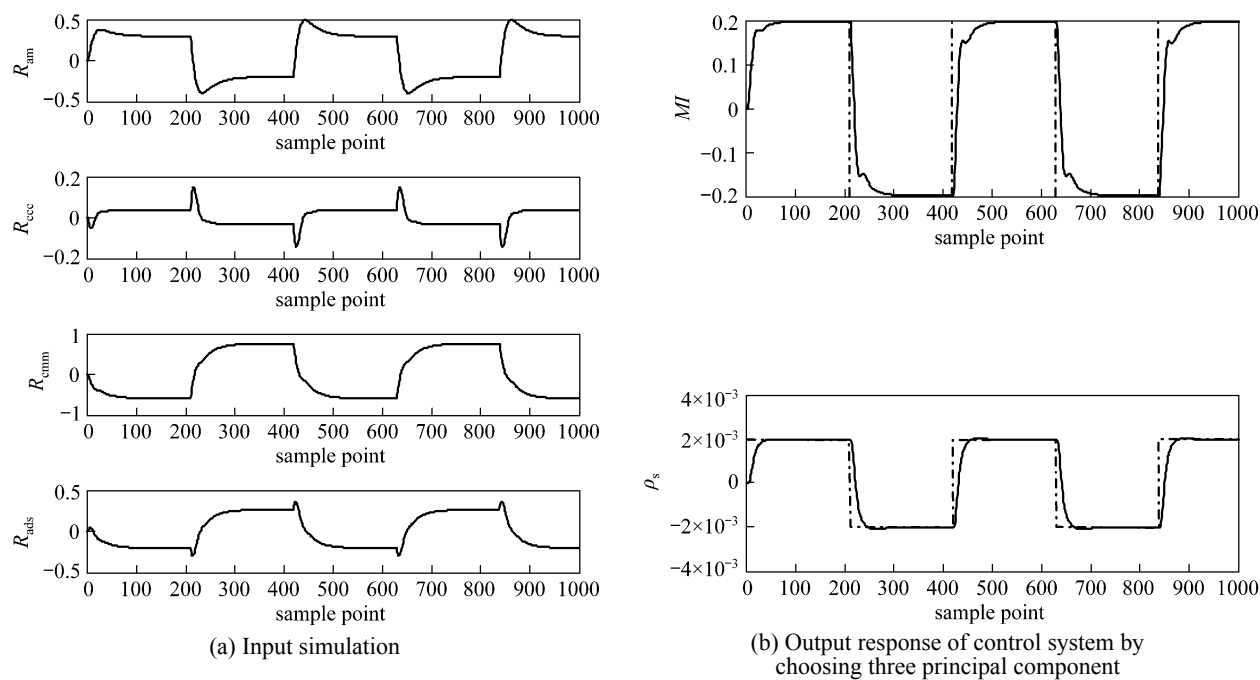


Figure 10 Response of system using IMC scheme in dimension reduction ARX-PLS framework

----- set points; — output of IMC controller

IMC controller [Fig.11 (a)], MPC controller can represents better tracking performance than IMC controller in ARX-PLS framework, and it can handle constraints problems of process variables. When system model structure such as the pole position changes, this method can maintain good control performance [Fig. 11 (b)] as adaptive IMC scheme proposed by Hu

to some extent.

5 CONCLUSIONS AND FUTURE WORK

In this paper, a multi-loop constraint MPC scheme based on ARX-PLS framework is proposed to

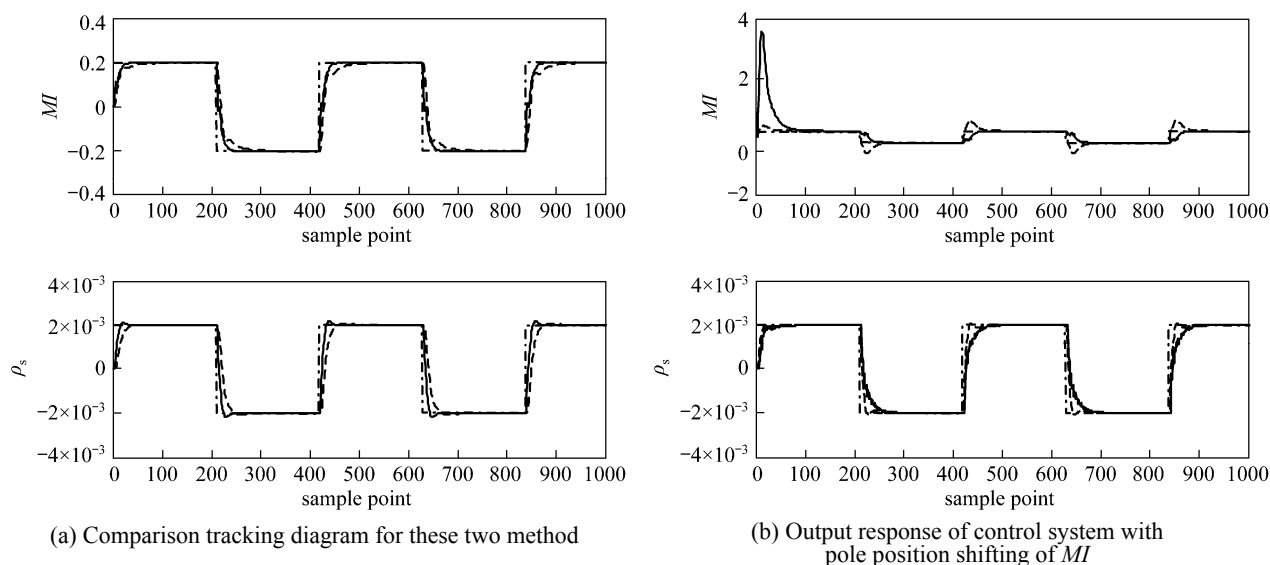


Figure 11 Comparison tracking diagram of system in using MPC controller and IMC controller in ARX-PLS framework
 - - - - set points; - - - output of IMC controller; — output of MPC controller

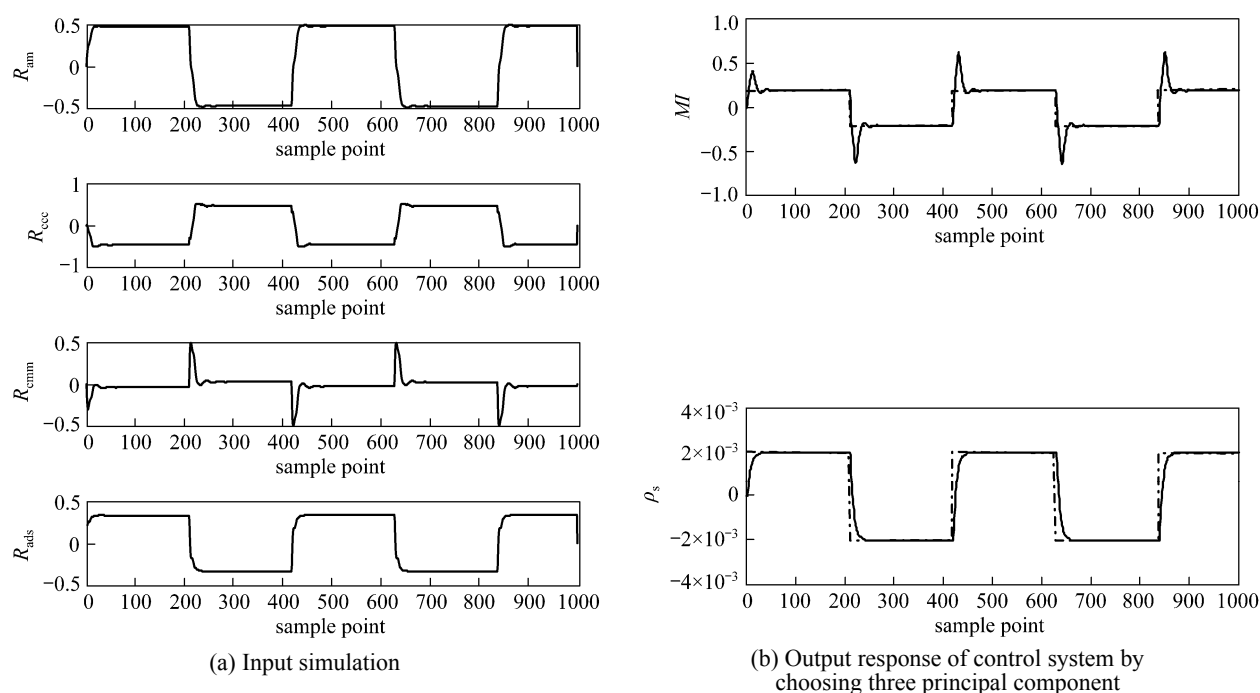


Figure 12 Response of system after turning the MPC parameters in dimension reduction ARX-PLS framework with the pole position of MI shifting from 0.4539 to 0.8851
 - - - - set points; — output of MPC controller

overcome the high dimension and constraints problems. The recursive least squares algorithm is used to identify the PLS model parameters, and an idea of iterative method is applied for the purpose of decoupling the constraints on MPC controller in latent variable subspaces. The study cases of non-square system and slurry reactor illustrate the effectiveness and robustness of the iterative constraint MPC method in ARX-PLS

framework.

However, although the disturbance such as white noise has been taken into consideration, the results have not satisfied the ideal control performance and the model-plant mismatch has not been deeply researched. Those problems will be considered by introducing another method such as robust MPC controller in the future.

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